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DEVELOPMENT OF A SABOTING TECHNIQUE FOR LIGHT GAS HYPERVELOCITY PROJECTORS

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San Ramon
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California
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DEVELOPMENT OF A SABOTING TECHNIQUE

FOR

LIGHT GAS HYPERVELOCITY PROJECTORS

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for

U. S. Naval Research Laboratory
Washington, D.C.

MB-R-63/14

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SUMMARY

A sabot, utilizing internal gas expansion to separate its halves, was developed for use in the launching of hypervelocity projectiles. Most sabots are designed so that aerodynamic forces will strip the parts away from the projectile. Here, separation was achieved by burning a propellant inside the sabot while it traveled down the gun barrel. Upon launch, the high pressure gases imparted lateral velocity to the halves by expansion work.

It was desired to achieve separation velocities of 1.5×10^4 cm/sec. At this velocity each half of the sabot would be displaced 10 cm from the trajectory at 6 meters from launch when fired at a velocity of 29,500 feet per second. A total of 33 experimental firings were carried out from CAL .60 powder and light gas guns. Both aluminum and fiberglas sabots were used. The lighter fiberglas sabot gave the desired separation velocity when fired at 8570 feet/second from the powder gun. However, it fractured when fired from the light gas gun. The aluminum sabot gave a lateral velocity of 0.8×10^4 cm/sec when fired at 14,000 feet/second from the light gas gun. The lower than predicted separation velocities may be a result of the assumptions used in the calculations. However, the velocities achieved are adequate for present light gas gun work.

Two major problems were high projectile dispersion and sabot mass. In most of the light gas gun firings the projectile missed the target. This was thought to be caused by the sabot geometry. Lower sabot masses are required to permit higher velocity firings. Improvements in materials and design should solve both the mass and dispersion problems.

I. INTRODUCTION

A current problem in hypervelocity projection is the technique of saboting the projectile. Sabots are necessary for achieving high velocity and for the acceleration of projectile shapes which are not suited to forming a gas seal within the launch tube. Once the acceleration is complete and the sabot and projectile assembly leaves the launch tube of the hypervelocity projector, the sabot must be separated from the projectile so that it will not interfere with in-flight and terminal ballistics measurements. The most widely used method of sabot separation at the present time is the application of the aerodynamic forces encountered in high velocity (even in the evacuated ranges). Thus the sabots are usually split into two or more pieces which have an undercut angled surface at the forward end. The air impinging upon the surface creates a lateral force component as well as a longitudinal one, and the sabot sections are pulled away from the projectile.

Under some circumstances, aerodynamic forces cannot be used to achieve sabot separation. For example, if the sabot must be made of a dense material, it may not move away from the projectile fast enough, particularly at very high velocities to prevent it from striking the target along with the projectile. This interferes with terminal ballistics data. Other situations would be where the sabot and projectile and design does not permit the use of aerodynamic separating surfaces.

The objective of this program is to develop a sabot which has an active rather than a passive means of separation. The method uses a propellant, such as gunpowder, which is burned in a cavity within the sabot while the sabot is in the launch tube of the hypervelocity projector. When the sabot leaves the launch tube, so that it is no longer restrained, the high pressure generated in the cavity will perform PdV work on the two sabot halves, causing them to fly apart. The propellant is ignited by a setback initiator at the instant the gun is fired. By adjusting the type and quantity of the propellant and the volume of the cavity, the total burning time can be controlled; thus, knowing the time the sabot will spend in the launch tube of any given gun, the propellant burn characteristics can be adjusted to match so that the maximum pressure is developed in the cavity (at the instant of complete propellant consumption) close to the time that the sabot leaves the launch tube.

The following criteria were established for this project:

1. The sabot is to be of aluminum, and will fit a caliber .60 launch tube. It will weigh 6 grams or less, exclusive of the projectile. Materials other than aluminum may be

investigated if they have potentially the same strength characteristics, but a lower density, so that the total mass may be reduced.

2. Each half of the sabot must be separated by 10 cm from the trajectory at 6 meters from a gun launch tube when fired at a velocity of 9 kilometers per second, and a range pressure of 1 mm Hg. The required velocity of separation of each sabot half is therefore 1.5×10^4 cm/sec.

3. The projectile is to be a 3/8" diameter steel sphere weighing 3.5 grams.

4. Testing during the development of the sabot will be done with a high velocity powder gun. Final tests will be done in a light gas gun.

II. DESIGN

A. REQUIREMENTS

The required kinetic energy of separation can be calculated simply from mechanics. Thus, for a separation velocity of 1.5×10^4 cm/sec and a maximum mass of 6 grams, the maximum energy required is

$$KE = \frac{mv^2}{2} = 67.5 \text{ joules}$$

For velocity ranges of interest, the acceleration time of most light gas guns is between 400 and 800 microseconds. Ideally, the propellant would be completely consumed close to the time the sabot emerges from the gun muzzle. This burning time requirement restricts the propellant to rifle and pistol powders.

Calculations were done for the following powders:

1) Bullseye

2) Unique

3) No. 2400

4) Hivel No. 2

5) Herco

All calculations assumed that 10 percent of the energy content of the powder would be converted to kinetic energy of the sabot halves. When computing the cavity volume for each powder, 10 percent was added for the initiator.

B. ENGINEERING CALCULATIONS

1. Calculation of PdV Work, Pressure, and Volume Change

The following procedure was used to compute the PdV work, pressure, and volume change requirements for the powders listed in Section A. (See Appendix A for detailed sample calculation.)

a. Using heat of explosion and 10 percent conversion efficiency, calculate mass of powder.

b. From bulk density and result of step a, calculate the volume the powder will occupy. Add 10 percent for initiator. This is propellant cavity volume.

c. Assuming the perfect gas law, $p = \frac{nRT}{V}$, calculate the pressure developed in the propellant cavity at complete consumption of the powder.

Assuming a reversible adiabatic expansion and a perfect gas, one may derive the following equation from thermodynamic relations:

$$W = \frac{\gamma nRT}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right] \tag{1}$$

where

W	=	required kinetic energy of sabot halves
γ	=	ratio of specific heats
n	=	moles of powder
R	=	gas constant
T	=	adiabatic, isochoric gas temperature
P_1	=	initial cavity pressure (maximum developed)
P_2	=	cavity pressure after performance of the required PdV work

d. Using given characteristics and results of steps a–c, calculate P_2 from Equation (1).

For a reversible adiabatic expansion, the following equation holds:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \tag{2}$$

where

P_1 = initial cavity pressure
P_2 = final cavity pressure
V_1 = initial cavity volume
V_2 = final cavity volume
γ = ratio of specific heats

e. Using the values determined above for P_1, P_2, V_1, and calculate V_2 from Equation (2). The values of V_1 and V_2 determined from this calculation procedure are input data for the mechanical design of the sabot propellant cavity.

The values of ΔP ($P_2 - P_1$) and ΔV($V_2 - V_1$) found from the above procedure were compared with those obtained directly from Newton's Second Law of mechanics, viz: $F = \frac{d}{dt} mv$.

The results thus obtained were found to be a reasonable check of the values calculated from thermodynamics.

2. Calculation of Powder Burn Time

Since the weights of powder, required cavity volumes, and ΔV for the five powders previously listed, were reasonable, the next step was to calculate the burning times. Using data supplied by the manufacturer, and calculated from the procedure described in Section 1, the burning times for these powders were computed as follows:

a. Using granulation data, calculate the volume of one grain.

b. Using the density and desired weight of powder, calculate the number of grains.

c. Using moles per grain data from manufacturer and result of step 6, calculate the total number of moles of gas produced by burning the powder.

Noting that the length of a grain is reduced by burning, along with the radius, it is possible to express the volume of a grain as a function of the radius. This equation is

$$V = \pi r^2(2r - K) = 2\pi r^3 - K\pi r^2$$

where

V = volume of powder per grain
r = radius of the grain
$2r - k$ = length of grain (k = constant depending on grain dimensions)

Now the number of moles of gas which can be produced, N', is

$$N' = m \rho V n^*$$

where

m = moles of gas per grain of powder
ρ = density of powder
n^* = number of grains

And the number of moles of gas present in the cavity at any given grain radius is

$$N(r) = N - N'$$

where

N = total number of moles of gas produced by burning all the powder, as found in step c.

$$N(r) = N - m\rho n^* (2\pi r^3 - K\pi r^2) \tag{3}$$

Since the pressure in the cavity is $P = \frac{RT}{V_1} N$, it can be expressed as a function of radius thusly,

$$P = \frac{RT}{V_1} N(r) \tag{4}$$

The burning rate of a powder grain is a function of the pressure

$$R = cP^n = \frac{dr}{dt} \tag{5}$$

Combining Equations (3), (4), and (5), we have

$$\frac{dr}{dt} = c\left\{\frac{RT}{V_1}\left[N - m\rho n^{*}(2\pi r^3 - K\pi r^2)\right]\right\}^n \qquad (6)$$

whose reciprocal is

$$\frac{dt}{dr} = \frac{1}{c}\left\{\frac{RT}{V_1}\left[N - m\rho n^{*}(2\pi r^3 - K\pi r^2)\right]\right\}^{-n} \qquad (7)$$

d. Using Equation (7), calculate $\frac{dt}{dr}$ for various values of r and plot $\frac{dt}{dr}$ vs. r.

e. Graphically integrate the curve obtained in step d. This value is the burning found to be in the range of interest.

The burning times of the five powders were calculated in this manner and found to be in the range of interest.

3. Results of Calculations

Results of the calculations are presented in Table 1. From this table it can be seen that propellants (1), (2), and (5) are suitable for use in light gas guns. Propellant (4) has a burning time suitable for the standard caliber 0.60 powder gun used for some of the experiments.

C. MECHANICAL DESIGN

1. Propellant Cavity and Cup

The accomplishment of PdV work which imparts kinetic energy to the sabot halves requires a discrete change of volume of the propellant cavity. Since each half of the sabot carries half of the propellant cavity, provision must be made for sealing in the high pressure gas to permit an adiabatic expansion as the sabot halves start to separate. If the propellant cavity is a cylinder this can be accomplished by simply placing a thin wall sleeve within the cavity. Thus the sabot halves slide along this sleeve for some small distance which allows

TABLE I

RESULTS OF CALCULATIONS

	(1)	(2)	(3)	(4)	(5)
Powder	Bullseye	Unique	2400	Hivel #2	Herco
Mass	124 mg	124 mg	150 mg	146 mg	152 mg
Cavity Volume (V_1)	0.245 cm^3	0.210 cm^3	0.192 cm^3	0.169 cm^3	0.284 cm^3
Pressure (P_1)	88 x 10^3 psi	99 x 10^3 psi	124 x 10^3 psi	136 x 10^3 psi	86 x 10^3 psi
Δ V (V_2 - V_1)	0.118 cm^3	0.105 cm^3	0.080 cm^3	0.074 cm^3	0.114 cm^3
Burn Time	554 μ sec	403 μ sec	1096 μ s	1200 μ s	664 μ s

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the proper volume expansion. There is a limit, of course, to the amount of travel that can be allowed, that is, the required PdV work would have to be completed when the sabot halves were some small distance apart. This separation distance was selected to be 0.1 inch.

Simply using a sleeve is not enough, however. It is quite likely that after separation the sleeve would remain behind the projectile and strike the target. Thus it is desirable to force the sleeve to separate with one of the sabot halves. This is easily accomplished by merely closing one end so that it takes the form of a cup. The gas pressure works against the closed end, transmitting the separating force through it to the sabot half so that the cup must go with that half.

The two different types of propellant cups used are shown in Figure 1. In Figure 1a is the propellant cup used with Hivel No. 2 powder. The cavity in which this cup was used had a volume of 0.169 cubic centimeters. These cups were fabricated from copper. The conical bottom is simply the angle of the end of a drill bit so that it fits properly in a drilled cavity. The cups were fabricated by punch and die. Figure 1b shows the cup used with 0.22 cm^3 and 0.25 cm^3 cavities. These cups were machined from 2024-T4 aluminum to reduce weight. The wall thickness however was made greater than that of the copper cup. This was necessary because the copper cup had a tendency to rupture under the high pressures developed in the cavities.

2. Sabot Design

The sabot configurations used were extensions of previous work. The three different types, all fabricated from 2024-T4 aluminum, are shown in cross section Figure 2. Figure 2a shows the Mark V sabot. This sabot was used only with the copper cup which is shown in Figure 1a. The conical tail is to prevent formation of a shock wave within the sabot to result from sudden application of the high pressure gas of the light gas gun. Figure 2b shows the Mark VI sabot. In order to reduce the weight of the sabot, metal was removed from the rear so that it was hemispherical instead of conical. It too used the copper cup of Figure 1a. Figure 2c shows the Mark VI Mod 1 sabot. This sabot was designed for the aluminum cup of Figure 1b and was used with the shorter burning time powders which require the larger cavity.

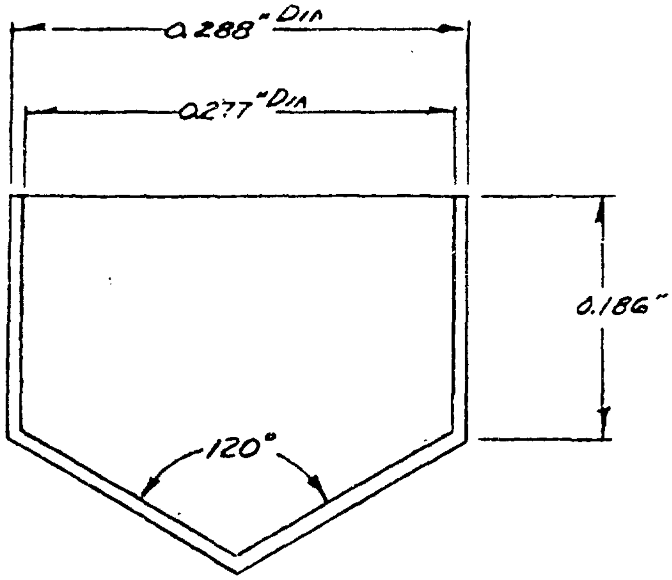

Figure 1(a) Copper Propellant Cup

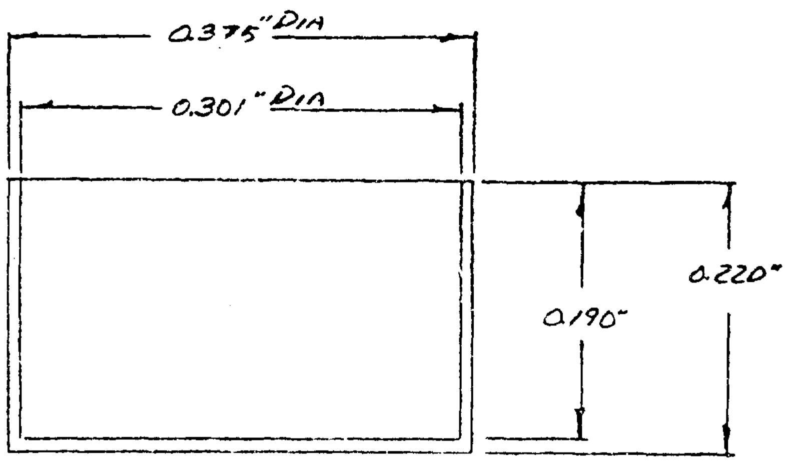

Figure 1(b) Aluminum Propellant Cup

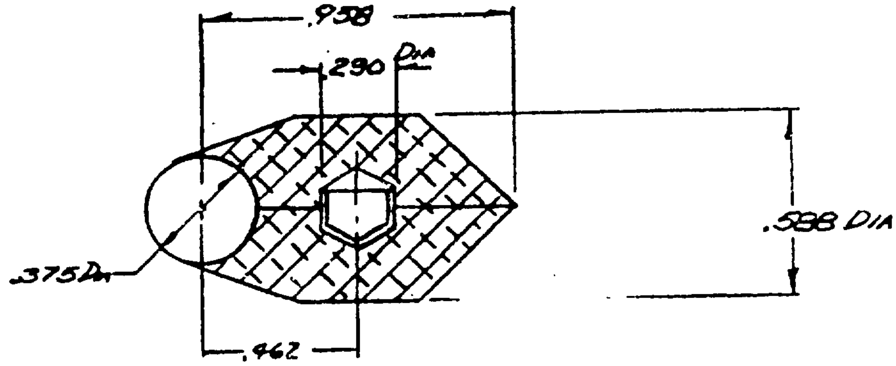

Figure 2(a) Mark V Sabot

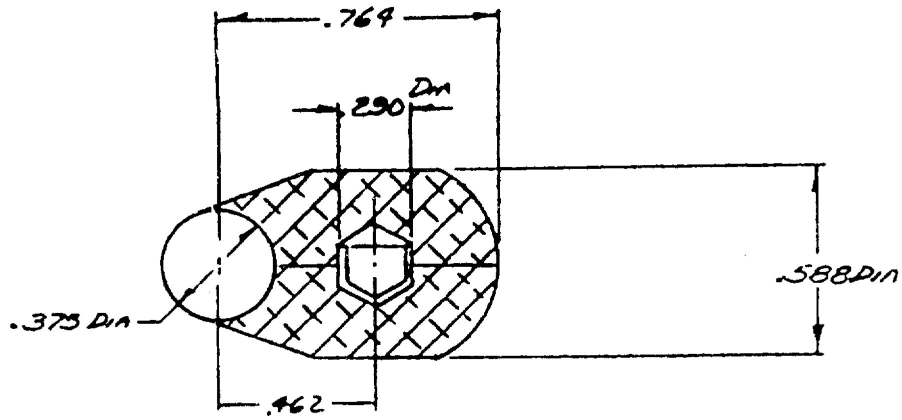

Figure 2(b) Mark VI Sabot

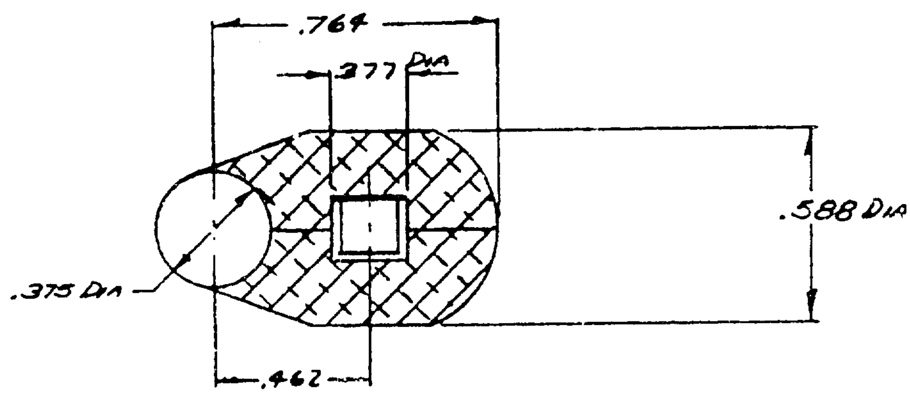

Figure 2 (c) Mark VI Mod 1 Sabot

One of the problems of designing sabots of this type is to keep the weight to a minimum. The presence of the cavity not only increases the required length of the sabot but also impairs its structural strength. The Mark VI Mod 1 sabot was designed to be made of aluminum. Its mass of approximately 9.85 grams was the minimum that could be obtained with any degree of structural integrity. In an attempt to reduce the mass by using a lower density material, an extensive search was made for plastics which might be strong enough to withstand the acceleration forces. Only one was selected for testing. This material was GB28E dielectroc fiberglass-epoxy laminate. The total mass of sabots made from this material was about 8 grams.

The projectile used in all of the sabots was a 3/8 inch diameter chrome alloy steel sphere weighing 3.52 grams. The mass of the sphere is included in the total masses given for the sabots. Also included in this total mass is the propellant and initiator.

3. Initiator

The initiator compound used to ignite the propellant was lead styphnate. This was packed into a thin walled glass tube which was placed in the propellant cavity. The tube was made by drawing down standard glass tubing to a small diameter. After resolidification of the glass, several initiator tubes could be cut from the drawn section.

Two kinds of tube placement were used. These are shown in Figure 3. During the first part of the experimentation, the tubes were placed as shown in Figure 3a, verticially in the rear of the cavity. In the remaining shots the tubes were placed as shown in Figure 3b, horizontally and longitudinally in the cavity, with one end of the glass tube against the front of the cavity. This latter arrangement was found to offer greater reliability.

The operation of the initiator is as follows. When the gun is fired the sudden acceleration causes the tube to crush under its own weight. The glass splinters are forced into the lead styphnate. The resulting friction detonates the styphnate which creates a very hot flash that ignites the propellant.

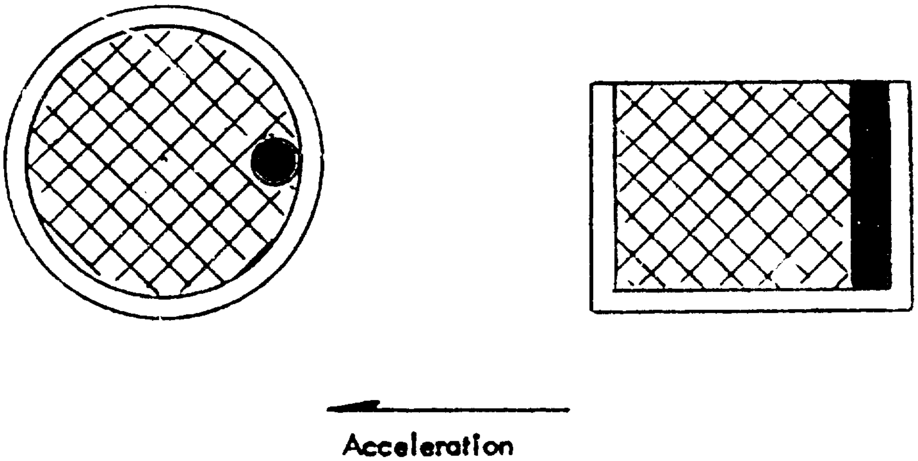

Figure 3(a) Vertical Initiator Placement

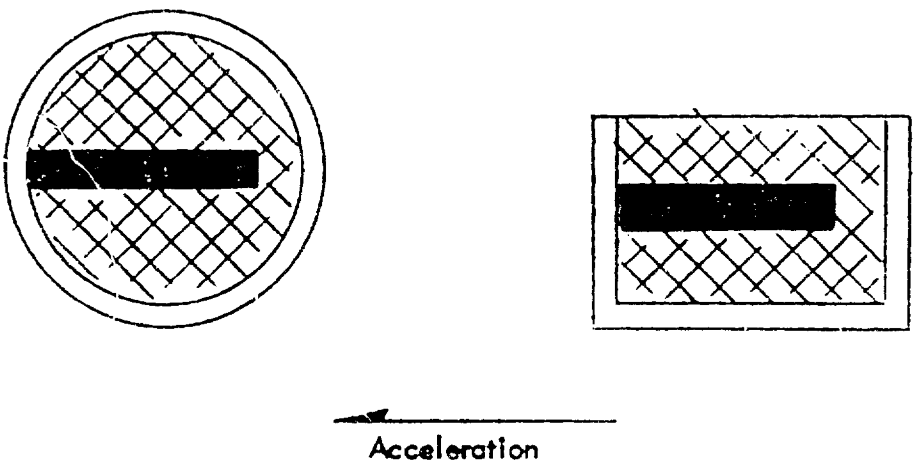

Figure 3(b) Horizontal-Longitudinal Initiator Placement

This initiator works quite well but was not entirely reliable. To improve reliability, finely ground glass was mixed with lead styphnate before packing into the tube. About one part glass was used with three parts of lead styphnate. These initiators proved to be entirely satisfactory. As an additional precaution, however, zirconium lead peroxide powder was mixed in with the propellant grains. This compound is much more flame sensitive than the propellant and puts out a hotter flash than the initiators. Thus it acted as a booster between the initiator and the propellant. Ten to twelve milligrams of lead styphnate-crushed glass mixture were used in each initiator and about 50 milligrams of zirconium lead peroxide $ZrPbO_2$ were used.

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III. EXPERIMENTS

Two types of experiments were conducted on this program and a total of 33 shots were fired. The development work was done with a caliber .60 powder gun range at velocities up to 8000 feet/second. This encompassed the first 27 shots. Final testing was done in the light gas gun range at the U. S. Naval Research Laboratory at velocities up to 14,000 feet/second.

A. EXPERIMENTAL FACILITIES

1. Powder Gun Range

A diagram of the powder gun range used is shown in Figure 4. Two smooth bore caliber .60 powder guns served as the high velocity projectors. One gun had a five foot long launch tube and was used for achieving maximum muzzle velocity. The other gun had a 2.5 foot long launch tube and was used for simulating the time-in-launch-tube of light gas guns so that the shorter burning time propellants could be tested.

The sabots were launched into a 12 foot long, 19 inch diameter vacuum chamber in which the separation took place. Inside the chamber were steel baffles spaced 16 inches apart down the full length. The projectile and sabots passed through 4 inch diameter holes in the center of the baffles. At the end of the tank was a 1/8 inch thick neoprene diaphragm 5 inches in diameter. After sabot separation and trapping by the baffles, the projectile passed through the rupture diaphragm into a sand chamber which was outside the tank. The gun was electrically fired by a 50 volt pulse from an electronic timing generator. This timing generator was also used for operation of the instrumentation. Overall control of the system was accomplished with a remotely operated sequence timer.

Instrumentation

Two types of diagnostics were used. The sabot and projectile velocity was measured by time of flight down the length of the range, and flash x-ray was used to investigate sabot separation. In addition the baffle assembly inside the tank was designed to be removed. Sabot separation velocity could therefore be measured from the locations of penetration of the baffles. A block diagram of the instrumentation system appears in Figure 5.

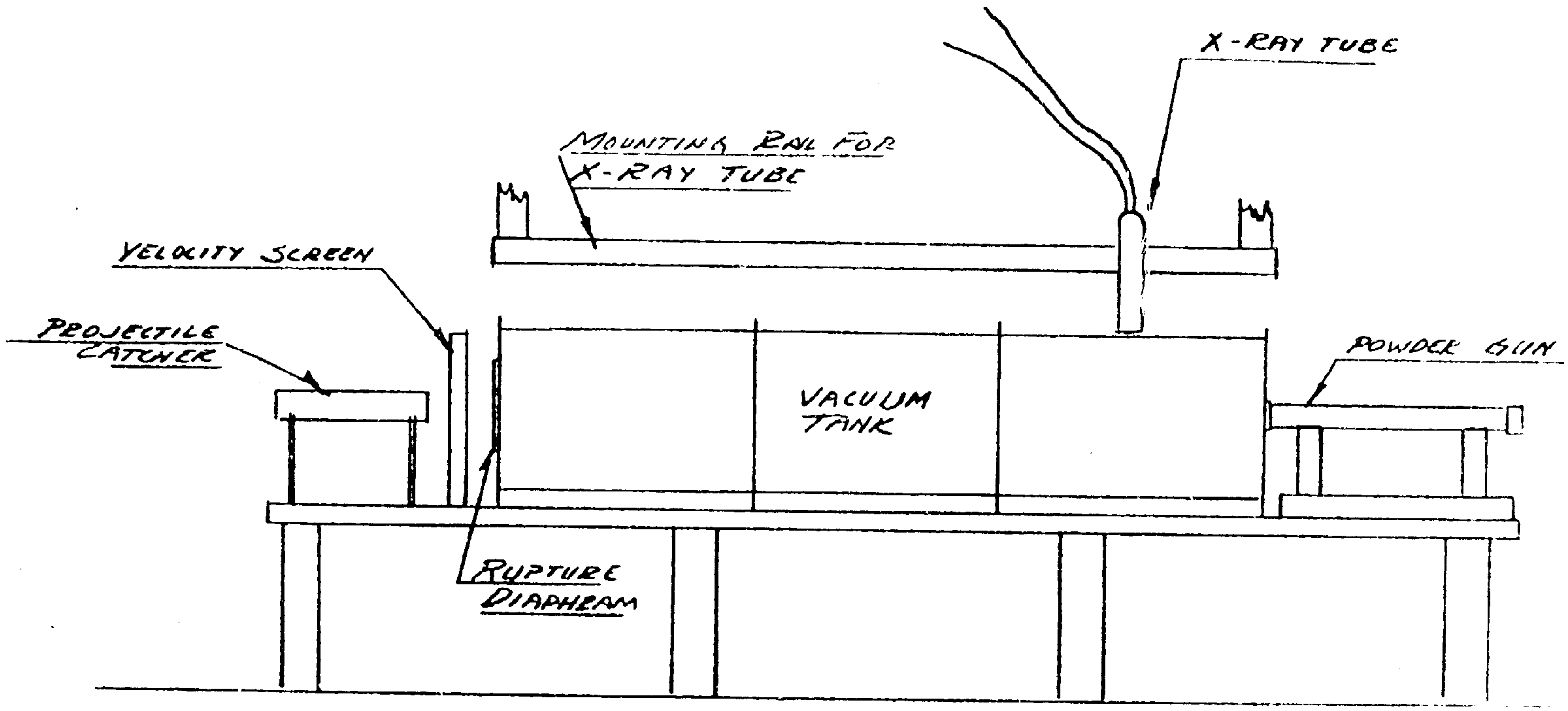


Figure 4. Diagram of Powder Gun Range

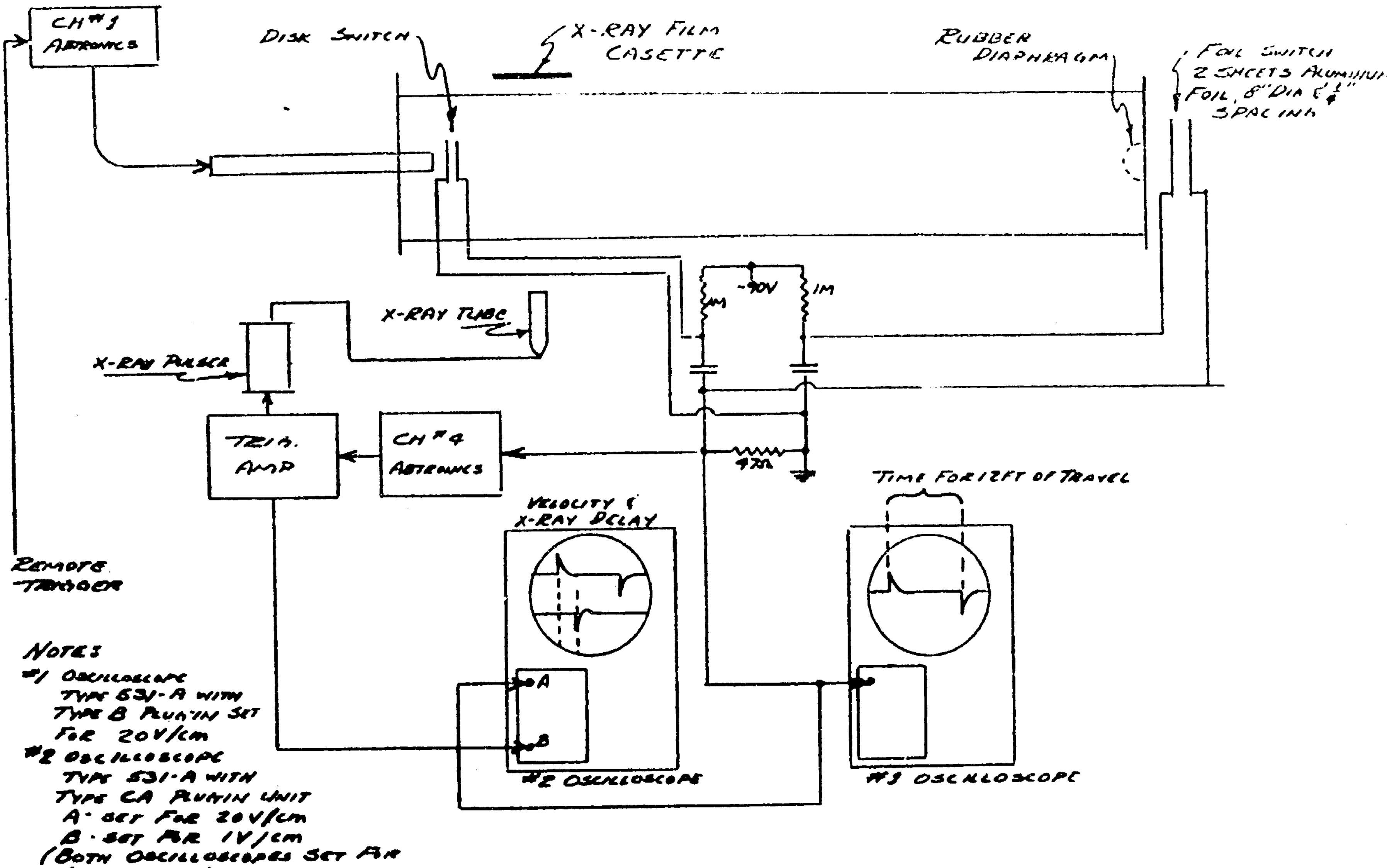


Figure 5. Block Diagram of Instrumentation System

Triggering of the flash x-ray and the first time of flight pulse was received from a switch at the end of the launch tube. Both make contact and break contact types were used which were actuated by the sabots. This switching proved to be the most troublesome part of the system. The percentage of reliability of proper functioning was much less greater than the desired.

The flash x-ray was a 1 channel, 105 kilovolt system with a 30 nanosecond pulse width. It was triggered by a time delay generator which was started by the switch at the end of the launch tube. The flash x-ray proved to be extremely valuable and much information was obtained. Failure of the triggering switch to function properly did prevent the acquisition of some data, although it was usually still possible to measure the sabot separation velocity from the baffle penetration.

The second time-of-flight signal was received from foil screens placed between the end of the vacuum chamber and the sand chamber which caught the projectiles. The sweep of an oscilloscope was started at the same time the firing signal was sent to the gun. The two pulses, one from the launch tube and the other from the time of flight screens, were then displayed on the oscilloscope sweep.

2. Light Gas Gun Facility

The last 6 experiments were conducted in a caliber .60 light gas gun at the U. S. Naval Research Laboratory, Washington, D. C. This gas gun uses a 40 millimeter pump tube and fires into an evacuated range approximately 35 feet in length. This vacuum range has thick steel baffles spaced down its entire length. Holes in the baffles are from 2 - 3 inches in diameter.

The instrumentation on these shots consisted of two high speed framing cameras. One of these was stationed at the target end of the range and was used to measure the velocity. The other was at the gun end of the range and was used to investigate sabot separation. Velocity measurements were made reliably, however, much difficulty was experienced with the sabot separation camera. This camera was stationed about 40 inches from the end of the launch tube. The gas stream from the launch tube was still dense enough at this position to obscure the projectile and sabot halves from view. The desired

range pressure was 1 millimeter of mercury. However, shots at 50 and 100 millimeters of mercury were tried in an attempt to slow down the gas stream. There was no improvement in this problem; however, some information was gathered from this camera station.

B. EXPERIMENTAL RESULTS

Table II is a summary of the experimental results. This table presents a cross section of the types of experiments that were done. The separation velocities achieved were not quite as high as desired, but are entirely adequate for most applications. For example: If the sabot can withstand the high "g" forces, a separation velocity of 1.0×10^4 cm/sec would give a total distance between sabot halves of about one foot at the end of a 35 foot range, for a projectile velocity of 25,000 feet/second. Figure 6 is the flash x-ray photograph showing separation of experiment 5.2-8. Figure 7 is a photograph of the sabot.

In general, it was found that if complete consumption of the cavity propellant (calculated burn time) occurred within 100 microseconds before sabot exit from the launch tube, the sabot separation velocity was not significantly impaired. If the time difference was 200 - 300 microseconds, the reduction in separation velocity was serious. This indicates that cavity pressure was lost due to cooling of the gas, or possibly leakage. Leakage seems unlikely because of the self-sealing effect of the gas pressure acting on the walls of the propellant cup. The aluminum propellant cups were found to be better than the copper cups. The copper had a tendency to rupture from the cavity pressure as the sabot halves separated.

The flash x-ray photographs showed that the sabots opened from the rear. This is probably due to the center of pressure being aft of the center of gravity of each sabot half. This may have induced some of the fracturing of the copper cups. It also could cause a lower separation velocity because of drag on the cup. The fiberglass sabots worked well in the powder gun. The one which was tried in the light gas gun broke up in the launch tube.

Of the six light gas gun shots, a separation velocity measurement was obtained on only one. This was 0.8×10^4 cm/sec. This was due to the difficulty with the instrumentation, as previously stated. Since the real test is not so much the separation velocity as it is just whether or not the sabot material strikes the target, steel and aluminum targets were used

TABLE II

SUMMARY OF EXPERIMENTAL RESULTS

Shot No.	Sabot Material	Powder	Vacuum	Velocity	Time in Launch Tube	Separation Velocity
5.2-8	Aluminum	Hivel #2	0.8 mm	7,800 fps	1,155 μ sec	1.1×10^4 cm/sec
5.2-11	Fiberglass	Hivel #2	0.6 mm	8,570 fps	1,155 μ sec	1.5×10^4 cm/sec
5.2-21	Aluminum	Bullseye	0.9 mm	6,850 fps	657 μ sec	1.0×10^4 cm/sec
5.2-27	Aluminum	Unique	0.8 mm	6,670 fps	675 μ sec	0.9×10^4 cm/sec
5.2-30	Aluminum	Bullseye	50 mm	14,000 fps	620 μ sec	0.8×10^4 cm/sec

All sabots Mark VI Model 1.
All projectiles 3/8" diameter chrome alloy steel sphere.

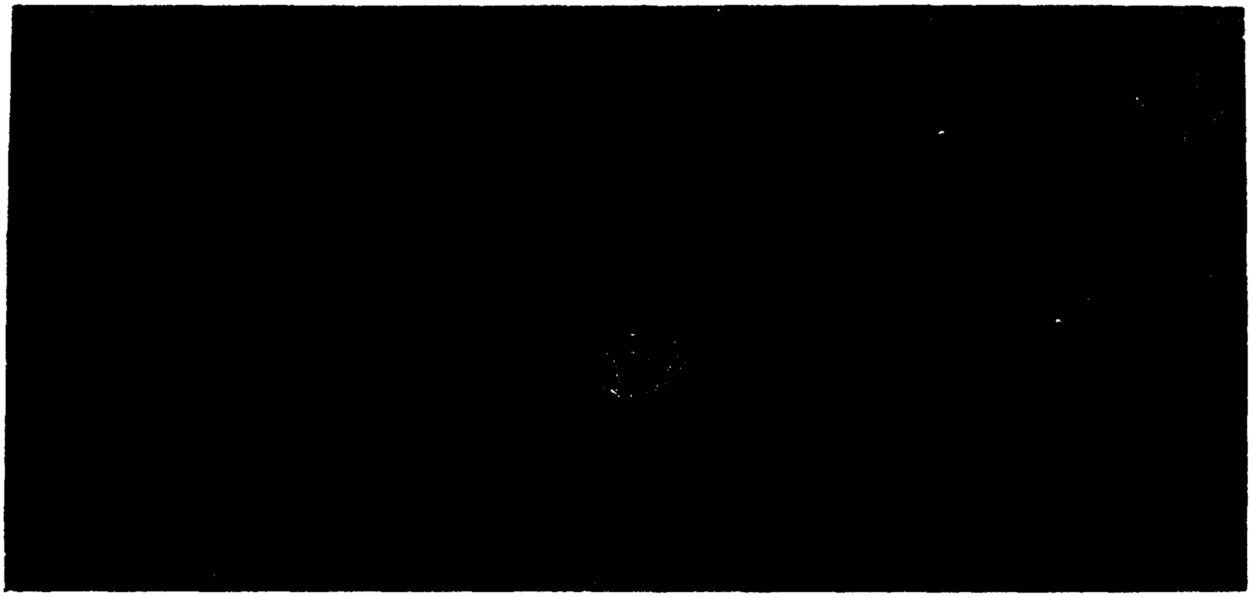
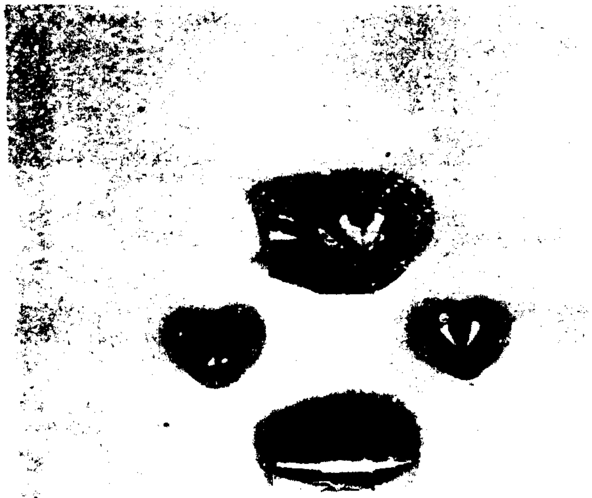
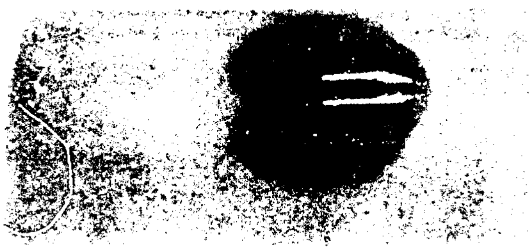


Figure 6. Flash X-Ray of Shot 5.2-8 Separation



Disassembled



Assembled

Figure 7. Photograph of Mark VI Mod. 1 Sabot

and examined after every shot. The first light gas gun shot fired was a dummy sabot, i.e., inert material was in the cup in place of the propellant. The intent of this was to look for breakup of the sabot in the launch tube. The sabot did not break up; the projectile struck the target, and a very few pieces of the sabot did also. On all of the other shots, however, the projectiles failed to hit the target. Some sabot material did strike the targets on most of these shots, however. Since the sabots apparently were not breaking up in the launch tube, and since separation was being achieved, it appears that some force was causing the projectile-sabot assembly to be knocked off trajectory. This appears to be substantiated by the fact that in several shots a part of the sabot struck the target but no projectile hit was observed. Thus if the whole assembly were canted out of line and separation was achieved, this result would be expected. The failure of the projectiles to hit the target in this shot could be attributed to one or more of three causes:

1 Balloting in the launch tube. This is quite likely, since the L/D of the sabot (contact surface) is less than 1. L/D of 2 is usually considered best.

2 Gas blast. The sabots begin to open immediately after leaving the launch tube. Since this opening is asymmetrical, the inside surfaces of the sabot halves are at different angles relative to the trajectory; thus a net lateral force could be applied by the gas stream emerging from the bore immediately behind the sabot. The force of the stream is an order of magnitude greater than from the powder gun.

3 Sabot-half dragging on projectile. Part way through the experimentation it was noticed that the projectiles in some of the remaining sabots had a tendency to stick in one half. This could cause the sphere to be pulled off trajectory as the sabot opened. Correction of this tendency to stick did not produce a target impact, however. It should be noted that the light gas gun range is quite long, with numerous baffles inside. Some of these baffles have holes only two inches in diameter, thus a small perturbation can prevent a hit.

Complete data on all the experiments conducted is given in Table III.

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	Sabot Type	Total Weight	Initiator Tube	Initiator Material	Powder
5.2-1	Mk V	11.02 gm	None	Pb styph.	Hivel
2	Mk V	11.02 gm	Vertical	"	"
3	Mk V	11.00 gm	"	"	"
4	Mk VI Mod 1	9.95 gm	"	"	"
5	Mk VI Mod 1	9.95 gm	"	"	"
6	M' VI Mod 1	9.95 gr	"	"	"
7	Mk VI Mod 1	9.95 gm	"	"	"
8	Mk VI Mod 1	9.85 gm	"	"	"
9	Mk VI Mod 1	9.85 gm	"	"	"
10	Mk VI Mod 1	9.85 gm	None	None	None
11	Mk VI Mod 1 fiberglass	8.09 gm	Vertical	Pb styph.	Hivel N
12	Mk VI Mod 1 fiberglass	8.09 gm	"	"	"
13	Mk VI fiberglass	8.09 gm	"	"	"
14	Mk VI	10.22 gm	Horizontal	"	"
15	Mk VI	10.1 gm	"	Pb styph./ crushed glass	"
16	Mk VI	10.16 gm	"	"	"
17	Mk VI Mod 1	10.09 gm	"	Styph. glass $ZrPbO_2$	"
18	Mk VI Mod 1 fiberglass	8.14 gm	"	"	"
19	Mk VI Mod 1 fiberglass	8.10 gm	"	"	"
20	Mk VI Mod 1 fiberglass	8.12 gm	"	"	Bullseye
21	Mk VI Mod 1	8.12 gm	"	"	"
22	Mk VI Mod 1	9.78 gm	"	"	"
23	Mk VI Mod 1	9.74 gm	"	"	Herco
24	Mk VI Mod 1	9.80 gm	"	"	Unique

TABLE III

EXPERIMENTAL RESULTS

Powder Type	Powder Weight	Powder Burn Time	Time in Launch Tube	Velocity	Vacuum	Separation Velocity	Ba
Hivel No.2	152 mg	1200 μs	1300 μs	6900 fps	0.8 mm	-	
"	"	"	1285 μs	7000 fps	0.8 mm	0.9×10^4 cm/sec	
"	"	"	1270 μs	7060 fps	0.4 mm	-	
"	"	"	1300 μs	6900 fps	0.8 mm	0.77×10^4 cm/sec	Y
"	"	"	1380 μs	6500 fps	0.2 mm	-	N
"	"	"	1340 μs	6700 fps	0.15 mm	0.74×10^4 cm/sec	Y
"	"	"	1300 μs	6930 fps	0.7 mm	-	N
"	"	"	1150 μs	7800 fps	0.8 mm	1.1×10^4 cm/sec	Y
"	"	"	1200 μs	7500 fps	0.8 mm	0.9×10^4 cm/sec	
None	-	-	1180 μs	7600 fps	0.9 mm	nil	N
Hivel No.2	152 mg	1200 μs	1150 μs	7800 fps	0.6 mm	1.5×10^4 cm/sec	Y
"	"	"	-	-	0.9 mm	-	N
"	"	"	-	-	0.9 mm	-	N
"	"	"	1160 μs	7750 fps	0.9 mm	0.52×10^4 cm/sec	Y
"	"	"	1160 μs	7750 fps	0.9 mm	0.26×10^4 cm/sec	Y
"	"	"	1180 μs	7650 fps	0.7 mm	0.62×10^4 cm/sec	Y
"	"	"	1160 μs	7750 fps	0.7 mm	0.04×10^4 cm/sec	Y
"	"	"	1120 μs	8000 fps	0.5 mm	1 Half 1.1×10^4 cm/sec	Y
"	"	"	-	-	0.9 mm	-	N
Bullseye	124 mg	554 μs	580 μs	7740 fps	0.8 mm	-	N
"	"	"	655 μs	6850 fps	0.9 mm	1.0×10^4 cm/sec	Ye
"	"	"	675 μs	6670 fps	0.7 mm	1.0×10^4 cm/sec	Ye
Herco	150 mg	664 μs	[illegible] μs	6670 fps	0.7 mm	0.07×10^4 cm/sec	No
Unique	124 mg	403 μs	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE III

EXPERIMENTAL RESULTS

elocity	Baffle	iator Material	Powder Type	Powder Weight	Powder Burn Time	Time in Launch Tube	Velocity	V
		b styph.	Hivel No.2	152 mg	1200 μs	1300 μs	6900 fps	(
	No	"	"	"	"	1285 μs	7000 fps	(
m/sec	Yes	"	"	"	"	1270 μs	7060 fps	0
	No	"	"	"	"	1300 μs	6900 fps	0
cm/sec	Yes	"	"	"	"	1380 μs	6500 fps	0
	No	"	"	"	"	1340 μs	6700 fps	0
m/sec	Yes	"	"	"	"	1300 μs	6930 fps	0
	No	"	"	"	"	1150 μs	7800 fps	0
/sec	Yes	"	"	"	"	1200 μs	7500 fps	0
/sec	-	ne	None	-	-	1180 μs	7600 fps	0.
	No	styph.	Hivel No.2	152 mg	1200 μs	1150 μs	7800 fps	0.
/sec	Yes	"	"	"	"	-	-	0.
	No	"	"	"	"	-	-	0.
	No	"	"	"	"	1160 μs	7750 fps	0.
n/sec	Yes	styph./ shed glass	"	"	"	1160 μs	7750 fps	0.
n/sec	Yes	"	"	"	"	1180 μs	7650 fps	0.
n/sec	Yes	ph. glass PbO_2	"	"	"	1160 μs	7750 fps	0.
/sec	Yes	"	"	"	"	1120 μs	8000 fps	0.
sec	Yes	"	"	"	"	-	-	0.9
	No	"	Bullseye	124 mg	554 μs	580 μs	7740 fps	0.8
	No	"	"	"	"	655 μs	6350 fps	0.9
sec	Yes	"	"	"	"	675 μs	6670 fps	0.7
sec	Yes	"	Herco	150 mg	664 μs	675 μs	6670 fps	0.7
/sec	No	"	Unique	124 mg	403 μs	675 μs	6670 fps	0.8

B

Vacuum	Separation Velocity	Baffle Impact	Sabot in Target	Remarks
0.8 mm	-	No	Yes	Initiator powder sprinkled in propellant.
0.8 mm	0.9×10^4 cm/sec	Yes	Little	
0.4 mm	-	No	Yes	
0.8 mm	0.77×10^4 cm/sec	Yes	No	X-ray shows sabot opening from the rear at 3" from launch tube.
0.2 mm	-	No	Yes	
0.15 mm	0.74×10^4 cm/sec	Yes	Very little	
0.7 mm	-	No	Yes	
0.8 mm	1.1×10^4 cm/sec	Yes	No	
0.8 mm	0.9×10^4 cm/sec	-	Very little	
0.9 mm	nil	No	Yes	Test for aerodynamic separation. X-ray shows sabot opening from front.
0.6 mm	1.5×10^4 cm/sec	Yes	Very little	
0.9 mm	-	No	No	
0.9 mm	-	No	Yes	
0.9 mm	0.52×10^4 cm/sec	Yes	Very little	X-ray shows cup ruptured.
0.9 mm	0.26×10^4 cm/sec	Yes	No	X-ray shows cup intact but free of sabot.
0.7 mm	0.62×10^4 cm/sec	Yes	Some	X-ray shows cup ruptured.
0.7 mm	0.04×10^4 cm/sec 1 Half	Yes	1 Half	
0.5 mm	1.1×10^4 cm/sec	Yes	Very little	
.9 mm	-	No	No	
.8 mm	-	No	Very little	
.9 mm	1.0×10^4 cm/sec	Yes	No	
.7 mm	1.0×10^4 cm/sec	Yes	No	
.7 mm	0.07×10^4 cm/sec	No	Yes	Front opening (aerodynamic) Powder failed to ignite.
.8 mm	nil	No	Yes	

	11	Mk VI Mod 1 fiberglass	8.09 gm	Vertical	Pb styph.	
	12	Mk VI Mod 1 fiberglass	8.09 gm	"	"	
	13	Mk VI fiberglass	8.09 gm	"	"	
	14	Mk VI	10.22 gm	Horizontal	"	
	15	Mk VI	10.1 gm	"	Pb styph./ crushed glass	
	16	Mk VI	10.16 gm	"	"	
	17	Mk VI Mod 1	10.09 gm	"	Styph. glass $ZrPbO_2$	
	18	Mk VI Mod 1 fiberglass	8.14 gm	"	"	
	19	Mk VI Mod 1 fiberglass	8.10 gm	"	"	
	20	Mk VI Mod 1 fiberglass	8.12 gm	"	"	
	21	Mk VI Mod 1	8.12 gm	"	"	
	22	Mk VI Mod 1	9.78 gm	"	"	
	23	Mk VI Mod 1	9.74 gm	"	"	I
	24	Mk VI Mod 1	9.80 gm	"	"	U
	25	Mk VI Mod 1	9.58 gm	"	"	E
	26	Mk VI Mod 1	9.83 gm	"	"	H
	27	Mk VI Mod 1	9.87 gm	"	"	L
light gas gun tests	28	Mk VI Mod 1	10.1 gm	None	None	N
	29	Mk VI Mod 1	9.91 gm	"	"	B
	30	Mk VI Mod 1	10.0 gm	"	"	
	31	Mk VI Mod 1	9.94 gm	"	"	
	32	Mk VI Mod 1 fiberglass	7.97 gm	"	"	
	33	Mk VI Mod 1	9.95 gm	"	"	

C

	Hivel No.2	152 mg	1200 μ s	1150 μ s	7800 fps	0.6 mm	1.5×10^4 cm/sec
	"	"	"	-	-	0.9 mm	-
	"	"	"	-	-	0.9 mm	-
	"	"	"	1160 μ s	7750 fps	0.9 mm	0.52×10^4 cm/se
ass	"	"	"	1160 μ s	7750 fps	0.9 mm	0.26×10^4 cm/se
	"	"	"	1180 μ s	7650 fps	0.7 mm	0.62×10^4 cm/se
ss	"	"	"	1160 μ s	7750 fps	0.7 mm	0.04×10^4 cm/se
	"	"	"	1120 μ s	8000 fps	0.5 mm	1 Half 1.1×10^4 cm/sec
	"	"	"	-	-	0.9 mm	-
	Bullseye	124 mg	554 μ s	580 μ s	7740 fps	0.8 mm	-
	"	"	"	655 μ s	6850 fps	0.9 mm	1.0×10^4 cm/sec
	"	"	"	675 μ s	6670 fps	0.7 mm	1.0×10^4 cm/sec
	Herco	150 mg	664 μ s	675 μ s	6670 fps	0.7 mm	0.07×10^4 cm/se
	Unique	124 mg	403 μ s	675 μ s	6670 fps	0.8 mm	nil
	Bullseye	124 mg	554 μ s	750 μ s	6000 fps	0.8 mm	0.9×10^4 cm/sec
	Herco	130 mg	664 μ s	675 μ s	6670 fps	0.9 mm	nil
	Unique	124 mg	403 μ s	675 μ s	6670 fps	0.8 mm	0.9×10^4 cm/sec
	None	-	-	910 μ s	13156 fps	1.0 mm	Yes
	Bullseye	124 mg	554 μ s	850 μ s	14133 fps	50.0 mm	Yes
	"	"	"	620 μ s	14000 fps	50.0 mm	0.8×10^4 cm/sec
	"	"	"	680 μ s	14000 fps	50.0 mm	Yes
	"	"	"	1000 μ s	12037 fps	100.0 mm	-
	"	"	"	-	-	1.0 mm	Yes

10^4 cm/sec	Yes	Pb styph.	Hivel No.2	152 mg	1200 μs	1150 μs	780
-	No	"	"	"	"	-	-
-	No	"	"	"	"	-	-
x 10^4 cm/sec	Yes	"	"	"	"	1160 μs	775
x 10^4 cm/sec	Yes	Pb styph./ crushed glass	"	"	"	1160 μs	775
x 10^4 cm/sec	Yes	"	"	"	"	1180 μs	765
x 10^4 cm/sec f	Yes	Styph. glass $ZrPbO_2$	"	"	"	1160 μs	775
10^4 cm/sec	Yes	"	"	"	"	1120 μs	800
	No	"	"	"	"	-	-
	No	"	Bullseye	124 mg	554 μs	580 μs	7740
10^4 cm/sec	Yes	"	"	"	"	655 μs	6850
10^4 cm/sec	Yes	"	"	"	"	675 μs	6670
x 10^4 cm/sec	No	"	Herco	150 mg	664 μs	675 μs	6670
l	No	"	Unique	124 mg	403 μs	675 μs	6670
10^4 cm/sec	Yes	"	Bullseye	124 mg	554 μs	750 μs	6000
l	No	"	Herco	130 mg	664 μs	675 μs	6670
10^4 cm/sec	Yes	"	Unique	124 mg	403 μs	675 μs	6670
es	-	None	None	-	-	910 μs	13156
es	-	"	Bullseye	124 mg	554 μs	850 μs	14133
10^4 cm/sec	-	"	"	"	"	620 μs	14000
es	-	"	"	"	"	680 μs	14000
	-	"	"	"	"	1000 μs	12037
es	-	"	"	"	"	-	-

7600 fps	0.9 mm	nil	No	Yes	Test for aerodynamic separation. shows sabot opening from front.
7800 fps	0.6 mm	1.5×10^4 cm/sec	Yes	Very little	
-	0.9 mm	-	No	No	
-	0.9 mm	-	No	Yes	
7750 fps	0.9 mm	0.52×10^4 cm/sec	Yes	Very little	X-ray shows cup ruptured.
7750 fps	0.9 mm	0.26×10^4 cm/sec	Yes	No	X-ray shows cup intact but free of sa
7650 fps	0.7 mm	0.62×10^4 cm/sec	Yes	Some	X-ray shows cup ruptured.
7750 fps	0.7 mm	0.04×10^4 cm/sec 1 Half	Yes	1 Half	
8000 fps	0.5 mm	1.1×10^4 cm/sec	Yes	Very little	
-	0.9 mm	-	No	No	
7740 fps	0.8 mm	-	No	Very little	
6850 fps	0.9 mm	1.0×10^4 cm/sec	Yes	No	
6670 fps	0.7 mm	1.0×10^4 cm/sec	Yes	No	
6670 fps	0.7 mm	0.07×10^4 cm/sec	No	Yes	Front opening (aerodynamic) Powder failed to ignite.
6670 fps	0.8 mm	nil	No	Yes	
6000 fps	0.8 mm	0.9×10^4 cm/sec	Yes	Very little	
6670 fps	0.9 mm	nil	No	Yes	Front opening (aerodynamic) Powder failed to ignite.
6670 fps	0.8 mm	0.9×10^4 cm/sec	Yes	No	
13156 fps	1.0 mm	Yes	-	Yes	Sabot break-up test. No break-up.
14133 fps	50.0 mm	Yes	-	1 Half	Projectile missed target.
4000 fps	50.0 mm	0.8×10^4 cm/sec	-	Very little	" "
4000 fps	50.0 mm	Yes	-	1 Half	" "
2037 fps	100.0 mm	-	-	Very little	" "
-	1.0 mm	Yes	-	No	" "

D

IV CONCLUSIONS

It can be seen from the experimental results that for velocities up to 14,000 fps this scheme of active sabot separation works. Although the separation velocity of the sabot halves was not as high as predicted by the calculations, it is nevertheless quite satisfactory for most applications.

It is difficult to assess the exact reasons for the separation velocity being less than calculated; however, the following factors may be involved:

1. The calculation assumed a reversible adiabatic expansion which this process is not. The error due to this is not expected to be greater than ten percent, however.

2. The assumed efficiency of energy conversion was ten per cent. This is believed to be a reasonable value, but a lower efficiency would have caused the slower velocity.

3. The burning time calculations are certainly not exact. A significant difference in the burning time and the time in launch tube would result in a lower velocity than calculated.

Additional experimental work would be required to determine which one of the above is causing the lower separation velocity. In all probability, it is some combination of the three. This is somewhat academic, however, since adequate separation performance was achieved.

V. RECOMMENDATIONS

Two problems remain which need additional attention. The first is the elimination of the effect which causes the projectiles to miss the targets in light gas gun ranges; the second is overall sabot weight reduction so that higher velocities may be achieved.

The two most likely effects which are causing the projectiles to miss the target are (1) sabot balloting in the launch tube, and (2) unsymmetrical opening of the sabot from the rear. The present L/D of the sabot is about 0.55. The L/D should be about 1.5 and preferably 2.0. With the present overall length of the sabot, an L/D of 1.5 could be achieved. This would require one of two things, either not turning a taper and a dome on the ends of the sabot so that the sabot becomes a cylinder, or the fitting of a hollow sleeve about the sabot which would extend the contact surface length without adding metal to the main sabot body. The first of these solutions has the drawback of adding a lot of weight. It would be however, a much more rigid structure. The addition of a hollow sleeve, as in the second suggestion, would require some kind of shoulder machined on the sabot to engage the sleeve and the sleeve itself would have to be fairly thin. There is some question whether this latter assembly would withstand the acceleration forces in the gun. Both should be tried, however.

The correction of the unsymmetrical opening of the sabot from the rear should not be a severe problem. The sabot tends to open from the rear now because the center of pressure from the cavity is aft of the center of gravity in the sabot halves. An attempt was made during this program to move the center of gravity aft by removing metal from the front taper of the sabot. This was unsuccessful, however.

The use of a cavity which is contoured more to the exterior shape of the sabot would probably correct this. Such a cavity could be any shape. These sabots would have to be made by casting. The use of casting would provide a possible solution to the problem of unsymmetrical opening, also. The unsymmetrical opening is now caused by the propellant cup going with one half of the sabot. Thus one half has a higher mass than the other. The use of casting would eliminate the need for a propellant cup. Interlocking ridges could be cast on each side of the sabot so that the gas seal is carried integrally with each half. It could be designed so that each half had the same amount of metal in the seal and thus the same mass. A contoured cavity would also permit a shorter sabot, reducing the overall mass

Significant mass reduction can only be accomplished by the use of lower density materials. At the moment this appears to be a severe problem. Many metallic and nonmetallic materials have been tried in various laboratories with no results so far. One possible solution however lies in the use of combinations of materials in a single sabot. For example, if it were desired to accelerate a high density projectile, the main body of the sabot could be made of a tough low density plastic material, such as Lexan, with a steel insert for carrying the projectile. The insert would be designed to distribute the force over a much larger area of the sabot than if the plastic were carrying the projectile directly. A program utilizing both design analysis and experimentation would be required to develop a configuration which would give the desired results.

APPENDIX A

SAMPLE CALCULATION FOR BULLSEYE PISTOL POWDER

Bullseye pistol powder has the following characteristics:

Solid density - $\rho = 1.6$ gm/cm^3

Bulk density - 0.56 gm/cm^3

Ratio of specific heats - $\gamma = 1.21$

Moles of gas generated - $m = 0.036 \dfrac{\text{gm moles}}{\text{gm (powder)}}$

Gas temperature (adiabatic, isochoric) - T = 4000° K

Heat of explosion - 1306 $\dfrac{\text{cal}}{\text{gm}}$

Granulation - 9.65×10^{-2} cm diameter x 8.38×10^{-3} cm long

Burn rate equation factors - $n = 1.006$
$c = 6.22 \times 10^{-4}$

1. Calculation of PdV Work, Pressure, and Volume Change

   a. Mass of Powder Required

$$M = 10 \times 67.5 \text{ joules} \times \frac{\text{cal}}{4.18 \text{ joules}} \times \frac{\text{gram}}{1306 \text{ cal}} = 124 \text{ milligrams}$$

   b. Volume Required

$$V = 124 \times 10^{-3} \text{ gram} \frac{\text{cm}^3}{0.56 \text{ gram}} = 0.222 \text{ cm}^3 = 13.58 \times 10^{-3} \text{ in}^3$$

add 10% for initiation

Cavity volume: $V_1 = 1.1 \times 0.222 \text{ cm}^3 = 0.245 \text{ cm}^3 = 14.88 \times 10^{-3} \text{ in}^3$

c. Cavity Pressure

Neglect contribution from initiator. Assume perfect gas law.

$$P_1 = \frac{nRT}{V} = \frac{124 \times 10^{-3} \text{ gm}}{2.45 \times 10^{-1} \text{ cm}^3} \times \frac{0.036 \text{ gm-moles}}{\text{gm}} \times \frac{8.2057 \times 10^{-2} \text{ liter atm}}{\text{gm-mole } ^\circ\text{K}} \times 4000^\circ \text{ K} \times \frac{10^3 \text{ cm}^3}{\text{liter}}$$

$$P_1 = 5990 \text{ atmospheres} = 88 \times 10^3 \text{ psi}$$

d. Pressure Change

Assume reversible adiabatic expansion. Assume perfect gas

$$W = -\frac{\gamma \, nRT}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$67.5 \text{ joules} = -\frac{(1.21)(124 \times 10^{-3} \text{gm})(0.036 \text{ gm-moles})(8.31 \text{ joules})(4000^\circ\text{K})}{(1.21-1) \qquad \text{gm} \qquad \text{gm-mole } ^\circ\text{K}}\left[\left(\frac{P_2}{88 \times 10^3}\right)^{\frac{1.21-1}{1.21}} - 1\right]$$

$$-\left(\frac{P_2}{88 \times 10^3}\right)^{0.1736} + 1 = 0.0789$$

$$\frac{P_2}{88 \times 10^3} = (0.9211)^{\frac{1}{0.1736}} = 0.622$$

$$P_2 = 54.75 \times 10^3 \text{ psi}$$

$$\Delta P = P_1 - P_2 = 33.25 \times 10^3 \text{ psi}$$

e. Volume Change

For reversible adiabatic expansion

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\frac{54.75 \times 10^3}{88 \times 10^3} = \left(\frac{0.245}{V_2}\right)^{1.21}$$

$$\frac{0.245}{V_2} = (0.622)^{\frac{1}{1.21}} = 0.6753$$

$$V_2 = 0.363 \text{ cm}^3 = 22.2 \times 10^{-3} \text{ in}^3$$

$$\Delta V = V_2 - V_1 = 0.113 \text{ cm}^3 = 7.3 \times 10^{-3} \text{ in}^3$$

2. Calculation of Powder Burn Time

a. Grain Volume

$$V' = \frac{\pi D^2 \ell}{4} = \frac{\pi (9.65 \times 10^{-2})^2 (8.38 \times 10^{-3})}{4} = 6.13 \times 10^{-5} \text{ cm}^3$$

b. Number of Grains

$$n^* = 124 \times 10^{-3} \text{ gram} \times \frac{\text{cm}^3}{.6 \text{ gram}} \times \frac{\text{grain}}{6.13 \times 10^{-5} \text{ cm}^3} = 1268$$

c. Total Moles of Gas Produced

$$N = mM = 3.6 \times 10^{-2} \frac{\text{m-moles}}{\text{gm}} \times 0.124 \text{ gm} = 4.46 \times 10^{-3} \text{ gm-moles}$$

d. <u>Calculate Burning Rate Vs. Grain Radius</u>

Use reciprocal of burning rate equation

$$\frac{dt}{dr} = \frac{1}{C} \left\{ \frac{RT}{V_1} \left[N - m \rho n^* (2\pi r^3 - k\pi r^2) \right] \right\}^{-n}$$

where $k = 2r - \ell = 9.65 \times 10^{-2} - 0.838 \times 10^{-2}$

$$= 3.812 \times 10^{-2} \text{ cm}$$

$$R = 1.205 \times 10^{3} \ \frac{\text{cm}^3 - \text{psi}}{\text{gm mole } ^\circ\text{K}}$$

$$\frac{dt}{dr} = \frac{1}{6.22 \times 10^{-4}} \left\{ \frac{1.205 \times 10^{3} \times 4 \times 10^{3}}{2.45 \times 10^{-1}} \right.$$

$$\left. \left[4.46 \times 10^{-3} - 3.6 \times 10^{-2} \times 1.6 \times 1.268 \times 10^{3} (2\pi r^3 - 8.812 \times 10^{-2} \pi r^2) \right] \right\}^{-1.006}$$

$$\frac{dt}{dr} = 1.61 \times 10^{3} \left\{ 1.97 \times 10^{7} \left[4.46 \times 10^{-3} - 2.29 \right. \right.$$

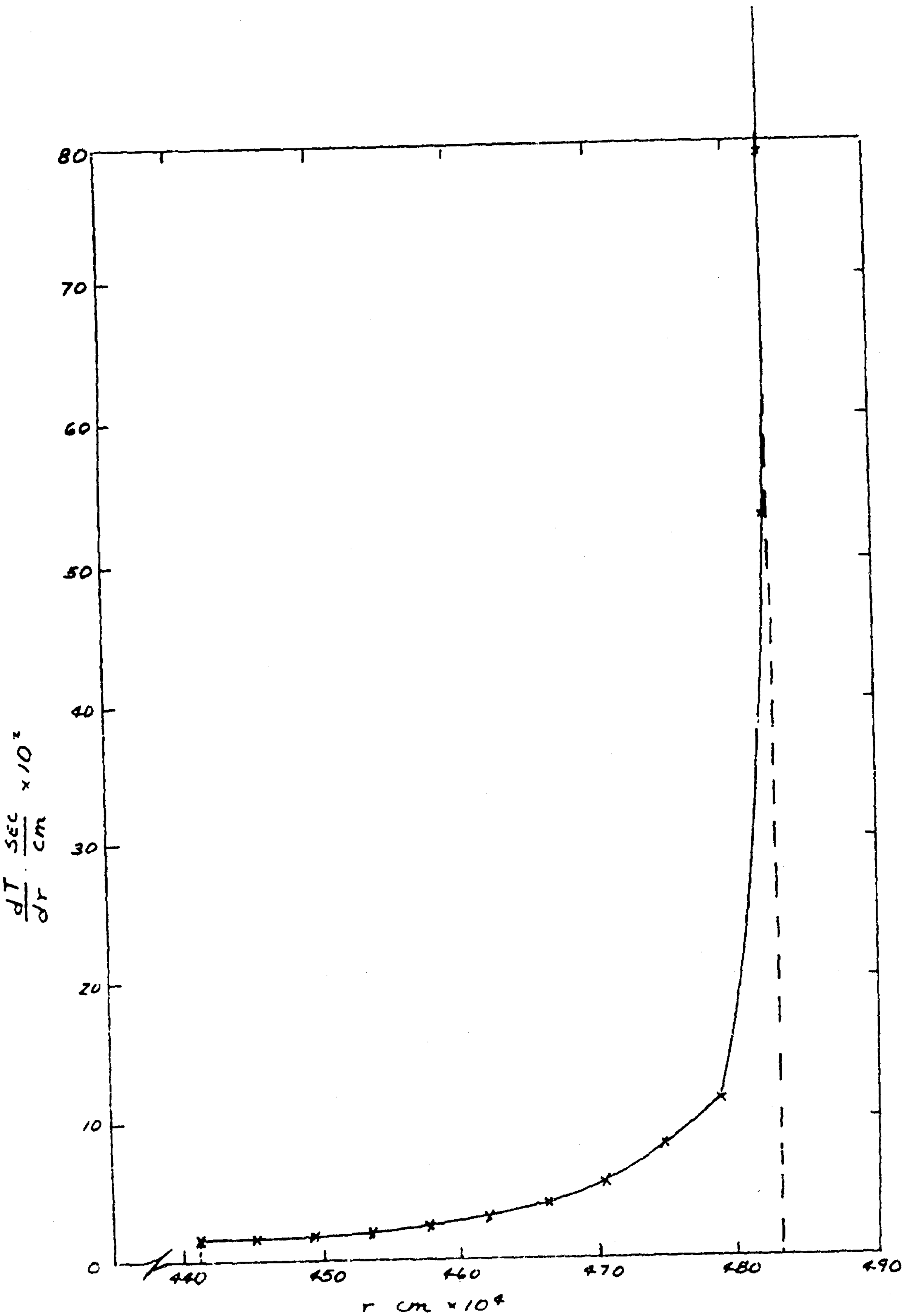
$$\left. \left. \times 10^{2} r^{2} (2r - 8.812 \times 10^{-2}) \right] \right\}^{-1.006}$$

From this equation, the following table lists $\frac{dt}{dr}$ and $\frac{dr}{dt}$ for various values of r.

r (cm)	$\frac{dt}{dr}$ $\frac{sec}{cm}$	$\frac{dr}{dt}$ $\frac{cm}{sec}$
0.04825		**0**
0.04824	**0.798**	**1.252**
0.04820	0.529	1.890
0.04800	0.224	4.460
0.04783	0.1128	8.87
0.04741	0.0830	12.05
0.04699	0.0560	17.85
0.04657	0.0415	24.10
0.04615	0.0326	30.70
0.04573	0.0275	36.39
0.04531	0.0233	42.95
0.04489	0.0211	47.40
0.04447	0.01905	52.50
0.04406	0.01778	56.30

Note that at r = 0.04406, the grain is completely consumed. This is because the length is much less than the diameter. Figure A is a plot of $\frac{dt}{dr}$ vs r.

e. Graphically integrating under the curve in Figure A, the burning time is found to be 5.54×10^{-4} seconds.



80
70
60
50
40
30
20
10
0
440
450
460
470
480
490
r cm ×10⁴
dT/dr sec/cm ×10²

Figure A